Hysteresis near transition of the large-scale dynamo in presence of the small-scale dynamo

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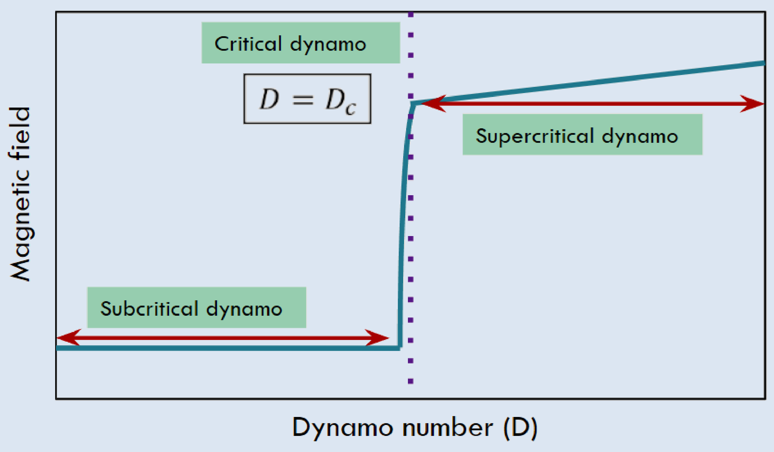
Abstract

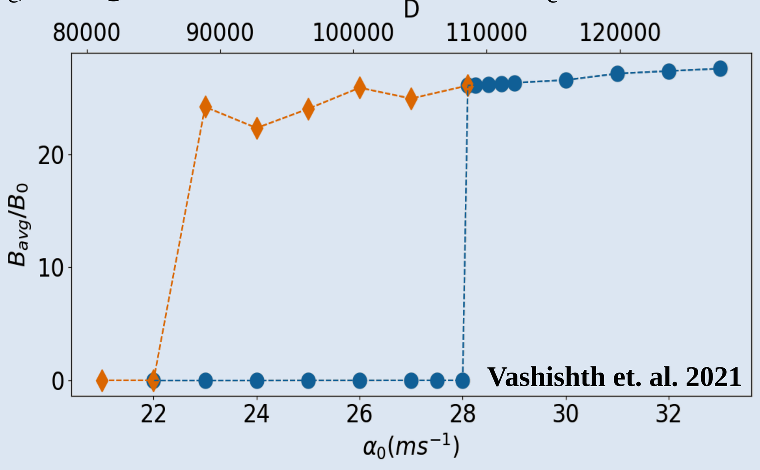
In the Sun, both large-scale (global) and small-scale (local) dynamos are expected to operate at the same time and location, we check the hysteresis behavior in a numerical model in which both large- and small-scale dynamos are excited. For this, we use the Pencil code and set up an αΩ dynamo model with uniform shear and helically forced turbulence. We have performed a set of simulations at different relative helicity to explore the generation of large-scale oscillatory fields in the presence of small-scale dynamo. We find that in some parameter regimes, the dynamo shows hysteresis behavior, i.e., two dynamo solutions are possible depending on the initial parameters used. A decaying solution was observed when the dynamo was started with a weak field and an oscillatory strong solution was seen if the dynamo was initialized with a strong field. Thus, the hysteresis of the large-scale dynamo is also observed in presence of the small-scale dynamo, however, the regime of hysteresis is quite narrow for the case without the small-scale dynamo.

Introduction

Solar dynamo: Large-scale:

- Sunspots, solar cycle and polar magnetic field are the manifestation of the large-scale magnetic field.
- This Large-scale magnetic field is produced by the large-scale solar dynamo that is operating in the solar convection zone.
- **Dynamo Hysteresis:** Crucial parameter that governs the dynamo is the **dynamo number,** $D = \frac{\alpha \Delta\Omega R^3}{\eta^2}$ Magnetic field decays for D below its critical value (D_c) and grows with time when $D > D_c$

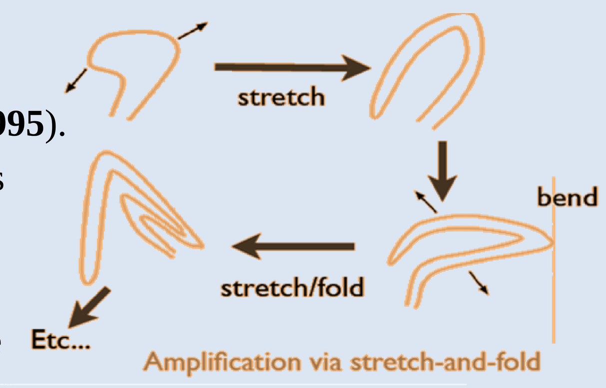


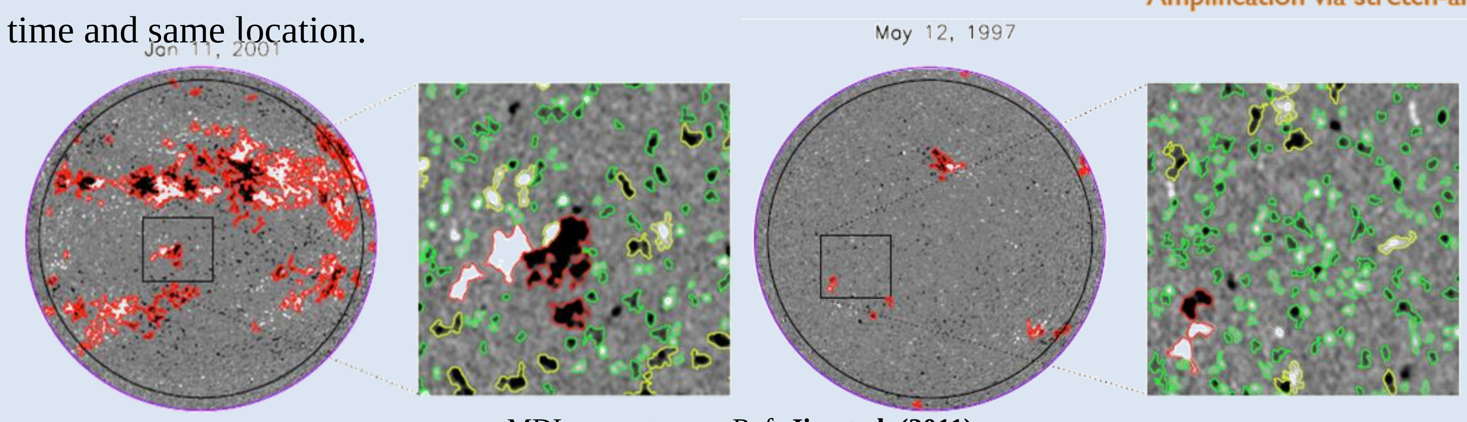


- In dynamo hysteresis, two dynamo solutions are possible depending on the initial parameters used.
- Previous studies of large-scale dynamo have found a hysteresis behavior near the dynamo transition **(Kitchatinov & Olemskoy 2010; Karak, Kitchatinov & Brandenburg 2015; Vashishth et. al. 2021)**.

Small-scale (local) dynamo:

- Dynamo processes exciting magnetic fields at scales smaller than the forcing scale of the flow.
- 3D velocity fields sufficiently random in space and/or time can amplify small-scale magnetic fluctuations via random stretching of the field lines (**Batchelor 1950; Zel'dovich et al. 1984; Childress & Gilbert 1995**).
- Small-scale dynamo works when the magnetic Reynolds number (R_m) is above a critical value.
- For the Sun, R_m is large.
- In the Sun, both global and local dynamos are expected to operate at the same time and same location.





MDI magnetogram Ref: **Jin et al. (2011)**

Motivation

- **Presence of small-scale dynamo is said to affect the presence of large-scale dynamo.** (**Karak & Brandenburg 2016, Bhat et. al. 2016**)
- **What is the hysteresis behavior of the large-scale dynamo in the presence of the small-scale dynamo.**

Model

αΩ dynamo model set up using PENCIL CODE (See, e.g. **Käpyla & Brandenburg 2009)**

- Isothermal and compressible fluid.
- It obeys the equation of state, $p = c_s^2 \rho$ with constant sound speed c_s^2
- Momentum Equation: $\frac{DU}{Dt} = \boldsymbol{f} - SU_x\hat{y} - c_s^2 \nabla \ln\rho + \frac{1}{\rho}[\boldsymbol{J}\times\boldsymbol{B} + \nabla\cdot(2\rho\nu\mathbf{S})]$, $D/Dt = \partial/\partial t + (\boldsymbol{U} + \boldsymbol{U}^{(S)})$

∇ = advective time derivative, $\boldsymbol{U}^{(S)} = (0, Sx, 0)$ = imposed uniform large-scale shear flow & S = constant shear rate

- Continuity equation: $\frac{D\ln\rho}{Dt} = -\nabla\cdot\boldsymbol{U}$,
- Induction equation: $\frac{\partial \boldsymbol{A}}{\partial t} + \boldsymbol{U}^{(S)}\cdot\nabla\boldsymbol{A} = -SA_y\hat{x} + \boldsymbol{U}\times\boldsymbol{B} + \eta\nabla^2\boldsymbol{A}$,
- In momentum equation, $\boldsymbol{f}$ = forcing function = helical and random in time (**Haugen et. al. 2004**),

$$\boldsymbol{f}(\boldsymbol{x}, t) = \mathrm{Re}\{N \boldsymbol{f}_{\boldsymbol{k}(t)} \exp[i\boldsymbol{k}(t)\cdot\boldsymbol{x} + i\phi(t)]\}$$

where, $\boldsymbol{f}_k = \mathsf{R}\cdot\boldsymbol{f}_k^{(\mathrm{nohel})}$ with $\mathsf{R}_{ij} = \frac{\delta_{ij} - i\sigma\epsilon_{ijk}\hat{k}_k}{\sqrt{1+\sigma^2}}$

σ = measure of the helicity of the forcing

$$\boldsymbol{f}_k^{(\mathrm{nohel})} = (\boldsymbol{k}\times\hat{\boldsymbol{e}}) \Big/ \sqrt{\boldsymbol{k}^2 - (\boldsymbol{k}\cdot\hat{\boldsymbol{e}})^2}$$

Results

I. Dynamo Transition

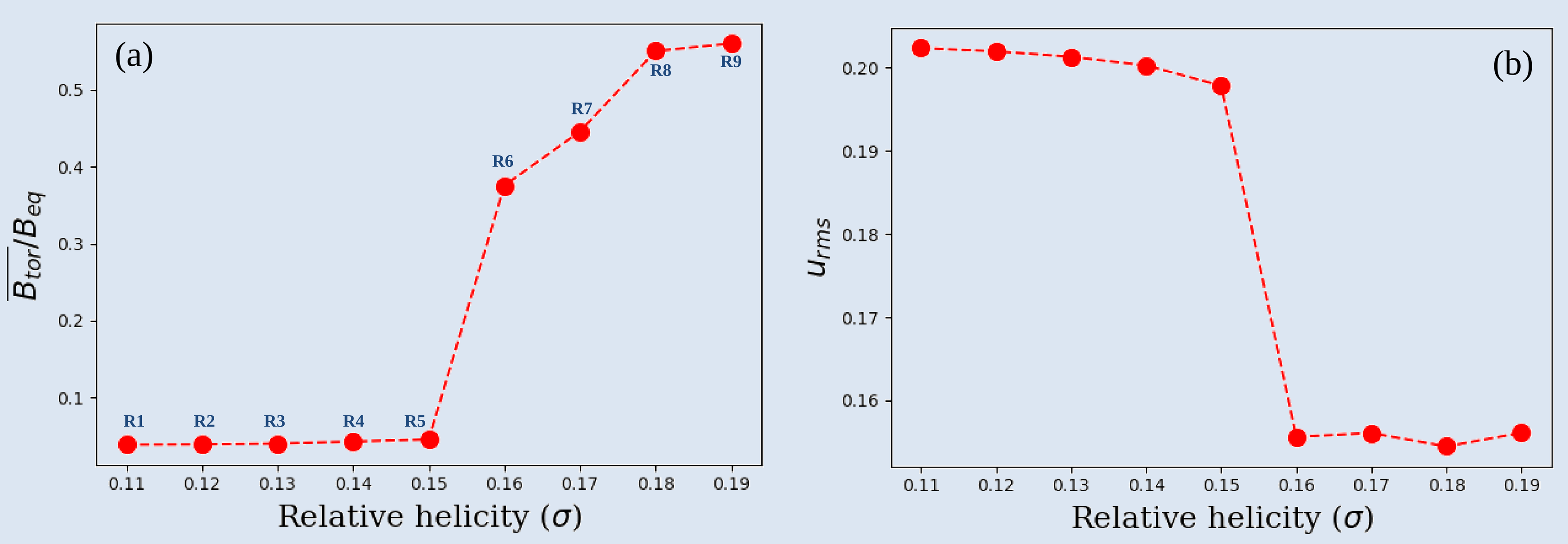


Figure 1: Variation of the (a) temporal average of the mean toroidal field and its (b) root mean square as a function of relative helicity from simulations started with a weak field.

Large-scale field:

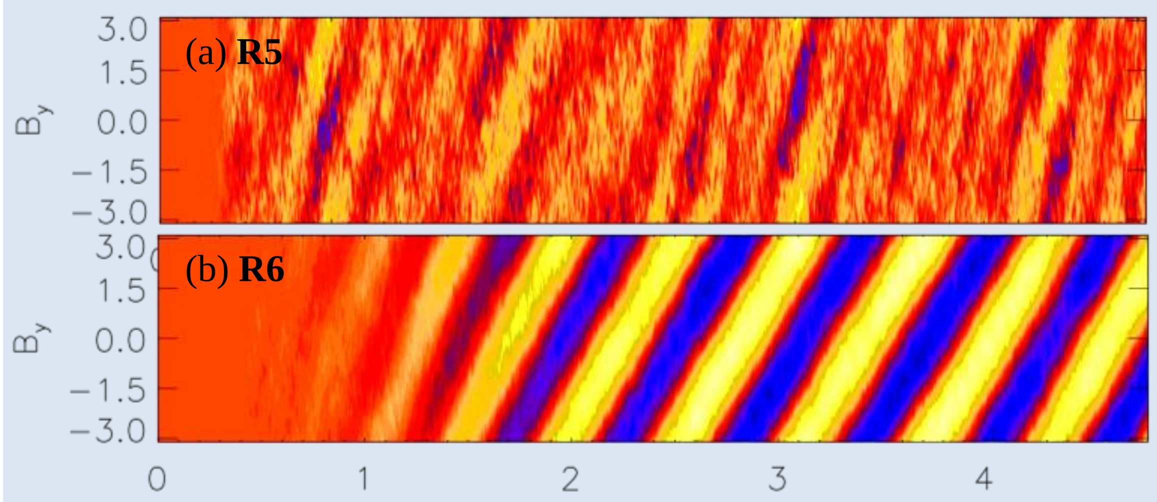


Figure 2: Butterfly diagrams of B_y as functions of z and t, normalized by the diffusive time scale for the

(a) Decaying resultant field at Subcritical dynamo regime , and

(b) Oscillating field for the Critical dynamo, when the simulation was started with a weak magnetic field.

Small-scale field in presence of large-scale:

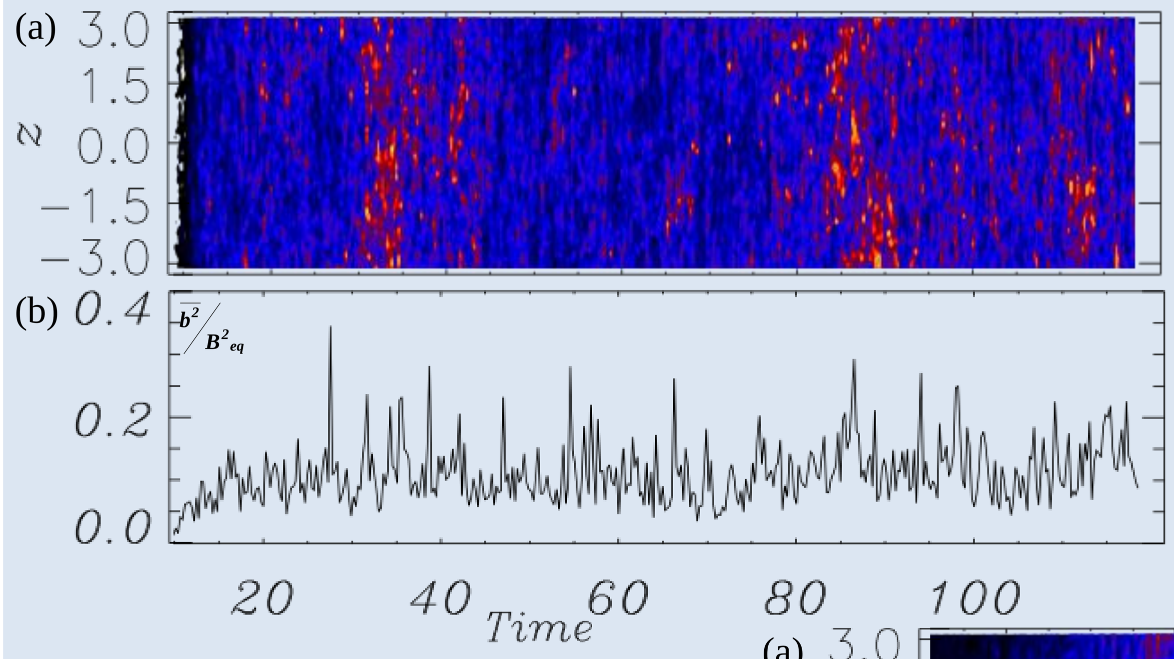


Figure 3:
(a) Butterfly diagram of the energy density of the large-scale magnetic field B^2, and
(b) time series of the small-scale field b^2 at z = 0 for the subcritical dynamo (R5).

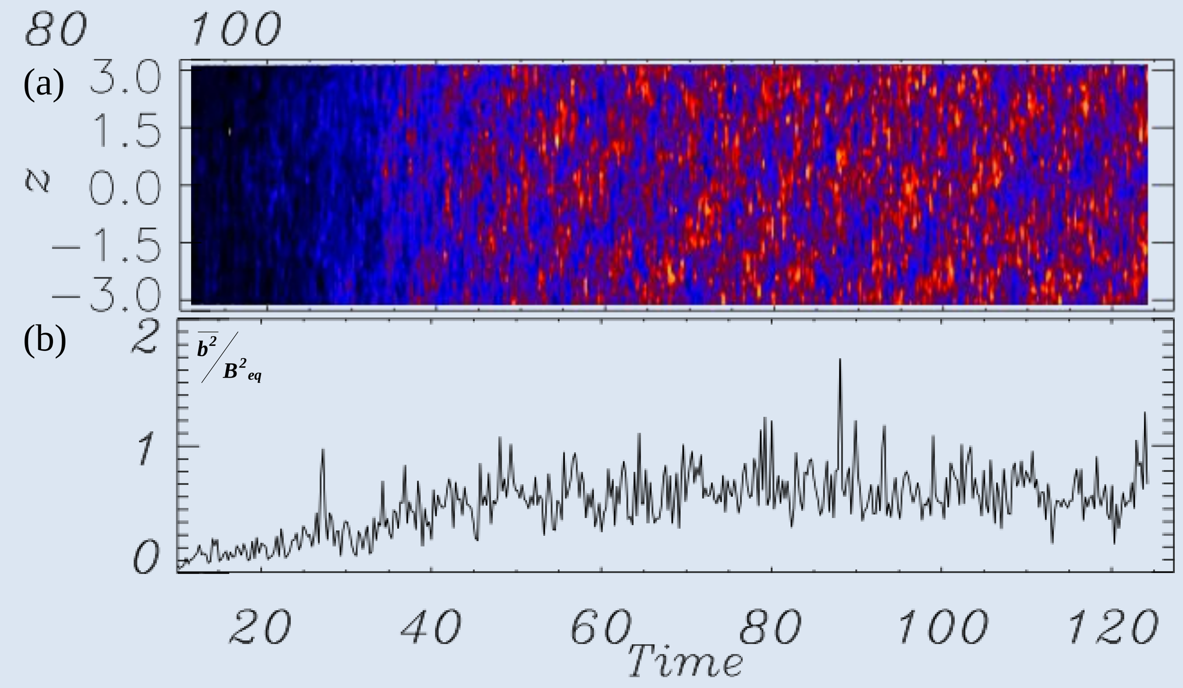


Figure 4:
Same as Figure 3, but for the critical dynamo regime (R6).

Figure 3 & 4 show the behavior of the small-scale field in presence of the large-scale field when the simulations are started with a weak magnetic field.

II. Dynamo Hysteresis

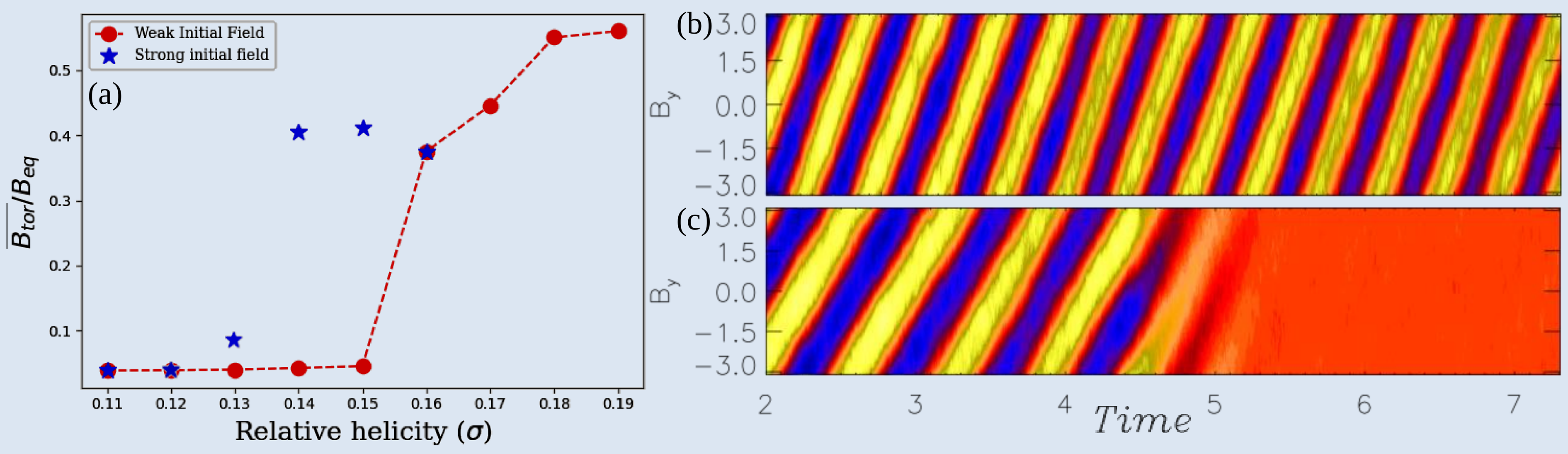


Figure 5: (a) : Variation of the temporal average of the mean toroidal field when from simulations started with a weak field. (red) and from simulations started with strong field of previous simulation (blue).
(b) Butterfly diagrams of B_y as functions of z and t, for the subcritical dynamos, σ = 0.15 and (c) σ = 0.12 when the simulation was started with a strong field of previous simulation.

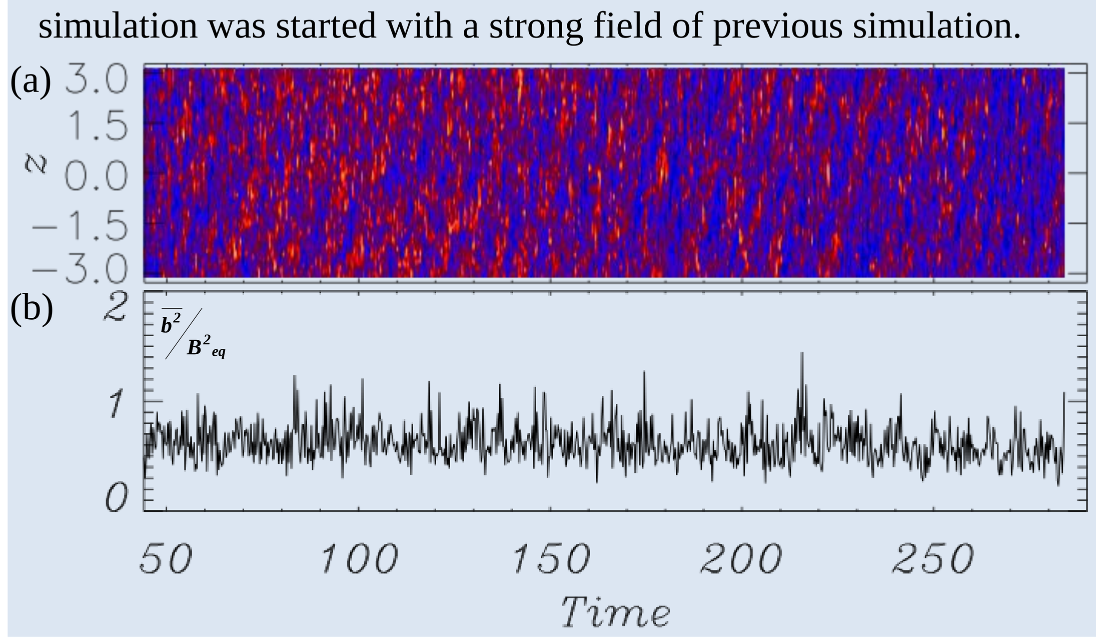


Figure 6: (a) Butterfly diagram of the energy density of the large-scale magnetic field B^2 and (b) time series of the small-scale field b^2 at z = 0 for the sub-critical dynamo σ = 0.15 when the simulation was started with a strong field of previous simulation.

Conclusions

- **With the help of PENCIL CODE, we have set up an αΩ dynamo simulation which excites both large-scale and small-scale dynamo.**
- **In this dynamo, fluid is helically forced and large-scale linear shear is imposed.**
- **We observed the dynamo transition and hysteresis of the large-scale dynamo in the presence of the small-scale dynamo.**

References

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